Homework 3

Due: Thursday 2/14/19 by 12:00pm (noon)

The R library polynom lets us easily compute the roots of polynomials. You'll need to install the polynom library and load it. I'll ask that you use it a bit in this homework, so here's a little example:

```r
library(polynom)

# Create a "polynomial" object for the polynomial
# 1 - 5x + 3x^2 + 2x^3
pol <- polynomial(c(1, -5, 3, 2))
# Get the values of x for which 1 - 5x + 3x^2 + 2x^3 = 0
sol <- solve(pol)
```

You may get complex roots $r = a + bi$. Note that the absolute value of a complex number r is given by $|r| = \sqrt{a^2 + b^2}$.

1. Consider the following $\mathbf{AR}(p)$ models, all with $\sigma_w^2 = 1$.

   i. $p = 1$, $\phi_1 = 0.99$
   ii. $p = 2$, $\phi_1 = 0.04$, $\phi_2 = 0.92$
   iii. $p = 2$, $\phi_1 = 0.04$, $\phi_2 = -0.92$
   iv. $p = 3$, $\phi_1 = 0.42$, $\phi_2 = -0.29$, $\phi_3 = 0.15$

   (a) For (i)-(iii), find the root of the autoregressive polynomial that is smallest in magnitude by solving $\phi(z) = 0$ for z by hand, without using any special R functions. For (iv), use polynom to find the root that is smallest in magnitude. Give the value of this root and indicate whether or not the model is causal.

   (b) Using ARMAacf to compute $\rho_x(h)$, plot the autocorrelation function $\rho_x(h)$ for $h = 0, \dots, 10$ for the causal $\mathbf{AR}(p)$ models on a single plot. Include a dotted horizontal line at 0.

   (c) Based on the plot you made in (b), describe what kinds of values of the autoregressive coefficients $\phi_1, \dots, \phi_p$ produce the following behaviors, using at most one sentence for each:

   - $\rho_x(h)$ oscillates between positive and negative values;
   - $\rho_x(h)$ oscillates between larger and smaller but always positive values;
   - $\rho_x(h)$ decays very quickly;
   - $\rho_x(h)$ decays very slowly.

2. Consider the following $\mathbf{MA}(q)$ models, all with $\sigma_w^2 = 1$.

   i. $q = 1$, $\theta_1 = 0.99$
   ii. $q = 2$, $\theta_1 = 0.04$, $\theta_2 = 0.92$
   iii. $q = 2$, $\theta_1 = 0.04$, $\theta_2 = -0.92$
   iv. $q = 3$, $\theta_1 = 0.42$, $\theta_2 = -0.29$, $\theta_3 = 0.15$

   (a) For (i)-(iii), find the root of the moving average polynomial that is smallest in magnitude by solving $\theta(z) = 0$ for z by hand, without using any special R functions. For (iv), use polynom to find the root that is smallest in magnitude. Give the value of this root and indicate whether or not the model is invertible.

   (b) Using ARMAacf to compute $\rho_x(h)$, plot the autocorrelation function $\rho_x(h)$ for $h = 0, \dots, 10$ for the invertible $\mathbf{MA}(q)$ models on a single plot. Include a dotted horizontal line at 0.

   (c) Based on the plot you made in (b) and the values you computed for the autocorrelation function $\rho_x(h)$, give the largest lag for which $|\rho_x(h)|$ is greater than zero for each $\mathbf{MA}(q)$ model.

3. Consider the following **ARMA**(p, q) models, all with $\sigma_w^2 = 1$.

i. $p = 1$, $q = 1$, $\phi_1 = 0.99$, $\theta_1 = 0.99$
ii. $p = 1$, $q = 2$, $\phi_1 = 0.99$, $\theta_1 = 0.04$, $\theta_2 = -0.92$
iii. $p = 2$, $q = 2$, $\phi_1 = 0.04$, $\phi_2 = 0.92$, $\theta_1 = 0.04$, $\theta_2 = 0.92$
iv. $p = 2$, $q = 2$, $\phi_1 = 0.04$, $\phi_2 = -0.92$, $\theta_1 = 0.04$, $\theta_2 = 0.92$
v. $p = 3$, $q = 3$, $\phi_1 = 0.42$, $\phi_2 = -0.29$, $\phi_3 = 0.15$, $\theta_1 = 0.42$, $\theta_2 = -0.29$, $\theta_3 = 0.15$

(a) Indicate whether or not each **ARMA**(p, q) model is causal, and indicate whether or not each **ARMA**(p, q) model is invertible.

(b) Using `ARMAacf` to compute $\rho_x(h)$, plot the autocorrelation function $\rho_x(h)$ for $h = 0, \dots, 10$ for the causal and invertible **ARMA**(p, q) models on a single plot. Include a dotted horizontal line at 0.

(c) Based on the plot you made in (b), describe what kinds of values of the autoregressive and moving average coefficients $\phi_1, \dots, \phi_p$ and $\theta_1, \dots, \theta_q$ produce the following behaviors, using at most one sentence for each:

- Quickly decaying correlations for small h and slowly decaying correlations for large h;
- Periodic/seasonal/cyclical behavior.